

Generating Neutrino Mass in the 331 Model

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Abstract

A mechanism for generating small tree-level Majorana mass for neutrinos is implemented in the 331 Model. No additional fermions or scalars need to be added, and no mass scale greater than a few TeV is invoked.

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1 Introduction

The recent atmospheric neutrino results from Super Kamiokande [1] have provided the long-awaited first direct evidence for physics beyond the Standard Model. The conventional interpretation of the results as due to neutrino oscillation requires neutrinos to have a very small, but non-zero mass of around 0.1 eV.

It is a challenge for theories to be able to explain why the neutrino mass should be so much smaller than that of all the other fermions, and a number of general mechanisms have been proposed. [2]

However, these methods generally require either the ad-hoc introduction of additional scalars, as in the Zee model [3] or other radiative mechanisms, or the presence of a very large mass scale, as in the seesaw mechanism [4] or the heavy Higgs triplet model. [5] While the latter two solutions are undoubtedly simple and very elegant, both require a new mass scale of some 10^{12} GeV - the mass of the right-handed neutrino in the seesaw model or the mass of the heavy Higgs in the triplet model. Apart from the fact that such a mass scale, being so widely separated from the electroweak breaking scale, introduces possible hierarchy problems, it is also displeasing in that it is hard to envisage how physics at such a high energy scale may be tested in the foreseeable future. [6]

Here we show that a method related to that of the Higgs triplet can be implemented with a minimum of contrivance within the popular 331 Model. [7]-[14] No heavy mass scale needs to be introduced, and in fact it is a feature of this model that in its minimal variant the symmetry breaking scale is actually constrained to be no more than a few TeV. The smallness of the neutrino mass is due to the small size of lepton-number violating terms in the scalar potential. We emphasise that no additional scalars or fermions need to be introduced.

2 The 331 Model

2.1 Fermions

The 331 Model is so-named because the Standard Model gauge group is extended to $SU(3)_c \times SU(3)_L \times U(1)_X$. The various versions which have been proposed can be characterised in part by the choice of the parameter ξ which describes the embedding of electric charge, according to

$$Q = \frac{\lambda_3}{2} + \xi \frac{\lambda_8}{2} + X \quad (1)$$

Here, we take $\xi = -\sqrt{3}$. The leptons of each family then transform in a triplet with $X = 0$, namely

$$f_{1,2,3L} = \begin{pmatrix} \nu_e \\ e^- \\ e^+ \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \\ \mu^+ \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \\ \tau^+ \end{pmatrix}_L \sim (1, 3, 0) \quad (2)$$

The first two families of quarks transform as $SU(3)_L$ anti-triplets and the third as a triplet.

$$Q_{1,2L} = \begin{pmatrix} d \\ u \\ D \end{pmatrix}_L, \begin{pmatrix} s \\ c \\ S \end{pmatrix}_L \sim (3, \bar{3}, -\frac{1}{3}) \quad (3)$$

$$Q_{3L} = \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L \sim (3, 3, +\frac{2}{3}) \quad (4)$$

All right-handed quarks transform as singlets. Note that D , S and T are exotic quarks, with charges $-\frac{4}{3}$, $-\frac{4}{3}$ and $\frac{5}{3}$ respectively. The above fermion representations are anomaly-free once all three generations are included.

2.2 Scalar Content

The minimal scalar content required to break symmetry and give all fermions a realistic mass consists of three triplets and one sextet:

$$\eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix} \sim (3, 0), \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (3, 1), \chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (3, -1) \quad (5)$$

$$S = \begin{pmatrix} \sigma_1^0 & s_2^- & s_1^+ \\ s_2^- & S_1^{--} & \sigma_2^0 \\ s_1^+ & \sigma_2^0 & S_2^{++} \end{pmatrix} \sim (6, 0) \quad (6)$$

These scalars have Yukawa couplings to the quarks as follows:

$$\begin{aligned} \mathcal{L}_{quarks}^{Yukawa} = & h_u^{ik} \bar{Q}_{iL} u_{kR} \rho^\star + h_d^{ik} \bar{Q}_{iL} d_{kR} \eta^\star + h_D^{ij} \bar{Q}_{iL} D_{jR} \chi^\star \\ & + h_d^{3k} \bar{Q}_{3L} d_{kR} \rho + h_u^{3k} \bar{Q}_{3L} u_{kR} \eta + h_T \bar{Q}_{3L} T_R \chi \end{aligned} \quad (7)$$

with $i, j = 1, 2$ and $k, l = 1, 2, 3$, while the Yukawa couplings of the leptons are given by

$$\mathcal{L}_{leptons}^{Yukawa} = h_A^{kl} \bar{f}_{kL} f_{lR}^\star \eta^\star + h_S^{kl} \bar{f}_{kL} S f_{lR}^\star \quad (8)$$

Written with explicit $SU(3)$ indices, these terms are of the form $\epsilon_{\alpha\beta\gamma} \bar{f}^\alpha f^{c\beta} \eta^{\star\gamma}$ and $\bar{f}^\alpha S_{\alpha\beta} f^{c\beta}$. The scalar multiplets gain VEVs as follows:

$$\langle \eta \rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}, \langle \rho \rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix}, \langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix} \quad (9)$$

and

$$\langle S \rangle = \begin{pmatrix} v'_4 & 0 & 0 \\ 0 & 0 & v_4 \\ 0 & v_4 & 0 \end{pmatrix} \quad (10)$$

The symmetry breaking scheme can be represented as:

$$SU(3)_L \times U(1)_X \xrightarrow{\langle \chi \rangle} SU(2)_L \times U(1)_Y \xrightarrow{\langle \eta, \rho, S \rangle} U(1)_Q \quad (11)$$

That is, the triplet χ is responsible for breaking $SU(3)_L \times U(1)_X$ symmetry, and therefore would receive a VEV of greater value than ρ , η and S , whose VEVs would be around the electroweak breaking scale. The exotic quarks and gauge bosons gain mass from v_3 , the Standard Model quarks gain mass from v_1 and v_2 , while the charged leptons gain mass from v_1 and v_4 . In the case of $v'_4 \neq 0$, neutrinos develop a tree-level majorana mass.

3 Global Symmetries

The Yukawa couplings written above possess three independent global symmetries which are not broken by the VEVs v_1, v_2, v_3 and v_4 . As well as the X charge already given we can assign conserved charges \mathcal{B} and \mathcal{L} to the fermion and scalar multiplets as follows:

$$\mathcal{B}(f_{1,2,3L}) = 0 \quad (12)$$

$$\mathcal{B}(Q_{1,2L}) = \mathcal{B}(Q_{3L}) = \frac{1}{3} \quad (13)$$

$$\mathcal{B}(u_{1,2,3R}) = \mathcal{B}(d_{1,2,3R}) = \frac{1}{3} \quad (14)$$

$$\mathcal{B}(D_{1,2R}) = \mathcal{B}(T_R) = \frac{1}{3} \quad (15)$$

$$\mathcal{B}(\eta) = \mathcal{B}(\rho) = \mathcal{B}(\chi) = \mathcal{B}(S) = 0 \quad (16)$$

and

$$\mathcal{L}(f_{1,2,3L}) = +\frac{1}{3} \quad (17)$$

$$\mathcal{L}(Q_{1,2L}) = +\frac{2}{3} \quad (18)$$

$$\mathcal{L}(Q_{3L}) = -\frac{2}{3} \quad (19)$$

$$\mathcal{L}(u_{1,2,3R}) = \mathcal{L}(d_{1,2,3R}) = 0 \quad (20)$$

$$\mathcal{L}(D_{1,2R}) = +2 \quad (21)$$

$$\mathcal{L}(T_R) = -2 \quad (22)$$

$$\mathcal{L}(\eta) = \mathcal{L}(\rho) = -\frac{2}{3} \quad (23)$$

$$\mathcal{L}(\chi) = +\frac{4}{3} \quad (24)$$

$$\mathcal{L}(S) = +\frac{2}{3} \quad (25)$$

The lepton and baryon number of the individual components of each multiplet will then be given by:

$$L = \frac{4}{\sqrt{3}} \frac{\lambda_8}{2} + \mathcal{L}I \quad (26)$$

$$B = \mathcal{B}I \quad (27)$$

4 The Scalar Potential

The full gauge-invariant scale potential was first given in Ref [12], and may be written as

$$V = V_{LNC} + V_{LNV} \quad (28)$$

where

$$\begin{aligned} V_{LNC} = & \mu_1^2 \eta^\dagger \eta + \mu_2^2 \rho^\dagger \rho + \mu_3^2 \chi^\dagger \chi \\ & + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\chi^\dagger \chi)^2 \\ & + \lambda_4 (\eta^\dagger \eta) (\rho^\dagger \rho) + \lambda_5 (\eta^\dagger \eta) (\chi^\dagger \chi) + \lambda_6 (\rho^\dagger \rho) (\chi^\dagger \chi) \\ & + \lambda_7 (\rho^\dagger \eta) (\eta^\dagger \rho) + \lambda_8 (\chi^\dagger \eta) (\eta^\dagger \chi) + \lambda_9 (\rho^\dagger \chi) (\chi^\dagger \rho) \\ & + f_1 (\eta \rho \chi + H.c.) \\ & + \mu_4^2 Tr(S^\dagger S) + \lambda_{10} [Tr(S^\dagger S)]^2 + \lambda_{11} Tr[(S^\dagger S)^2] \\ & + [\lambda_{12} (\eta^\dagger \eta) + \lambda_{13} (\rho^\dagger \rho) + \lambda_{14} (\chi^\dagger \chi)] Tr(S^\dagger S) \\ & + f_2 (\rho \chi S^\dagger + H.c.) \\ & + \lambda_{15} \eta^\dagger S S^\dagger \eta + \lambda_{16} \rho^\dagger S S^\dagger \rho + \lambda_{17} \chi^\dagger S S^\dagger \chi \\ & + \lambda_{19} \rho^\dagger S \rho \eta + \lambda_{20} \chi^\dagger S \chi \eta + \lambda_{21} \eta \eta S S + H.c. \end{aligned} \quad (29)$$

and

$$\begin{aligned} V_{LNV} = & \bar{f}_3 \eta S^\dagger \eta + \bar{f}_4 S S S + \bar{\lambda}_{22} \chi^\dagger \eta \rho^\dagger \eta + \\ & \bar{\lambda}_{23} \eta^\dagger S \chi \rho + \bar{\lambda}_{24} \chi \rho S S + H.c. \end{aligned} \quad (30)$$

The coefficients f_1, f_2, \bar{f}_3 and \bar{f}_4 have dimensions of mass, and bars have been used to denote terms which do not conserve lepton number, \mathcal{L} , as defined above.

In previous studies, [10, 15] the lepton number violating terms have often been excluded, commonly by the adoption of an appropriate discrete symmetry. While there is no reason within the 331 Model why such terms should not be present, experimental limits on processes which do not conserve total lepton number such as neutrinoless double beta decay, [16] require them to be very small.

If the potential is restricted to V_{LNC} , then two solutions exist which minimise the potential and satisfy equations 9 and 10 above, namely $v'_4 = 0$ or

$$\begin{aligned} 2\lambda_{11} v_4'^2 = & -(\lambda_{15} + 2\lambda_{21}) v_1^2 + \frac{\lambda_{16}}{2} v_2^2 + \frac{\lambda_{17}}{2} v_3^2 + \lambda_{11} v_4^2 \\ & - \frac{\lambda_{19}}{\sqrt{2}} \frac{v_1 v_2^2}{v_4} + \frac{\lambda_{20}}{\sqrt{2}} \frac{v_1 v_3^2}{v_4} + \frac{f_2}{\sqrt{2}} \frac{v_2 v_3}{v_4} \end{aligned} \quad (31)$$

The latter solution leads to the formation of majorons (since lepton number is now being broken spontaneously) and may be excluded by LEP. [17]

If the full potential is used, on the other hand, a VEV for v'_4 is automatically induced [18] to satisfy the constraint

$$Av_4'^3 + \bar{B}v_4'^2 + Cv_4' + \bar{D} = 0 \quad (32)$$

where

$$A = 2\lambda_{11} \quad (33)$$

$$\bar{B} = 6\bar{f}_4 + \sqrt{2}\bar{\lambda}_{24}\frac{v_2v_3}{v_4} \quad (34)$$

$$C = (\lambda_{15} + 2\lambda_{21})v_1^2 - \frac{1}{2}(\lambda_{16}v_2^2 + \lambda_{17}v_3^2) - \lambda_{11}v_4^2 \\ + \frac{1}{\sqrt{2}}\frac{v_1}{v_4}\left(\lambda_{19}v_2^2 - f_2\frac{v_2v_3}{v_1} - \lambda_{20}v_3^2\right) \quad (35)$$

$$\bar{D} = \bar{f}_3v_1^2 - 3\bar{f}_4v_4^2 - \bar{\lambda}_{23}v_1v_2v_3 - \sqrt{2}\bar{\lambda}_{24}v_2v_3v_4 \quad (36)$$

that is,

$$v_4' = \frac{-\bar{D}}{C + \bar{B}v_4' + Av_4'^2} \quad (37)$$

In other words, $\langle\sigma_1^0\rangle \simeq -\bar{D}/C$. From equation 8, $m_{\nu_\tau}/m_\tau \sim v_4'/v_4$ and assuming $v_4 \sim 100$ GeV, to obtain $m_{\nu_\tau} \simeq 0.1$ eV requires $\bar{D}/C \simeq 6 \times 10^{-9}$ GeV. Note that this ratio is consistent with other experimental constraints. Apart from neutrinoless double beta decay mentioned earlier, v_4' is also required to be small to maintain a value for the ρ -parameter sufficiently close to unity. [19]

It is perhaps worth pointing out that, although this ratio might seem large, ratios of large order already exist in the 331 Model in the Yukawa sector. As another comparison, in the Standard Model, the ratio of the top quark mass to the electron mass of 3.4×10^5 is due entirely to the relative strength of the respective Yukawa couplings.

5 Conclusion

We have shown that a mechanism for generating very small tree-level neutrino masses can easily be implemented in the 331 Model.

Lepton number violation occurs very naturally in these models since the charged lepton of each family and its antiparticle are placed in the same triplet, and with the one assumption that lepton-number violating terms in the scalar potential are sufficiently small, tree-level majorana neutrino masses of order 0.1 eV are automatically induced without the introduction of any additional fermions or scalar multiplets to the model. We stress that no mass scale greater than a few TeV needs to be made use of, and thus this model remains testable at the next generation of colliders.

We also note briefly that, as an alternative, a similar mechanism can also be employed in a second variant of the 331 Model, in which a heavy lepton is

included in the lepton triplet. The leptons of each family then transform as follows:

$$\begin{pmatrix} l^- \\ \nu_l \\ L^+ \end{pmatrix}_L \sim (1, 3, 0), l_R^- \sim (1, 1, -1), L_R^+ \sim (1, 1, +1) \quad (38)$$

where l^- is the conventional charged lepton of each family and L^+ represents an exotic heavy lepton, both of which now obtain Dirac masses from the triplets ρ and χ respectively.

An interesting consequence of this version of the model is that bileptons only couple standard to exotic leptons and thus many processes which have been studied in order to place lower limits on bilepton mass will not occur.

In this scenario, the sextet can be made very heavy with only very small values for v_4 and v'_4 induced. However, since a new mass scale $\mu_4 \sim 10^{12}$ GeV must be introduced, we do not find this method as attractive as the main one outlined.

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